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Random Magnetic Impurities and the δ Impurity Problem

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Abstract. — One considers the effect of disorder on the 2-dimensional density of states of an electron of charge e in a constant magnetic field superposed onto a Poissonnian random distribution of point vortices carrying a flux ϕ ($\alpha = e\phi/2\pi$ is the dimensionless coupling constant). If the electron Hilbert space is restricted to the Lowest Landau Level (LLL) of the total average magnetic field, the random magnetic impurity problem is mapped onto a contact δ impurity problem. Particular features of the average density of states are then interpreted in terms of the microscopic eigenstates of the N impurity Hamiltonian. The deformation of the density of states with respect to the density of impurities manifests itself by the progressive depopulation of the LLL. A Brownian motion analysis of the model, based on Brownian probability distributions for arithmetic area winding sectors, is also proposed. In the case $\alpha = \pm 1/2$, the depletion of states at the bottom of the spectrum is materialized by a Lifschitz tail in the average density of states.

1. Introduction

One considers a 2-dimensional model for an electron of electric charge e and of mass m subject to a constant external magnetic field B superposed onto a random distribution $\mathbf{r}_i$, $i = 1, 2, ..., N$ of fixed infinitely thin vortices carrying a flux ϕ, modeling magnetic impurities, and characterized by the dimensionless Aharonov-Bohm (A-B) coupling $\alpha = e\phi/2\pi = \phi/\phi_0$. Particular interest will be paid to the effect of disorder on the energy level density $\rho(E)$ of the test particle averaged over the random position of the vortices [1]. In the thermodynamic limit $N \to \infty, V \to \infty$ for a distribution of vortices of density $\rho = N/V$, the average magnetic field, $< B >= \alpha\rho\phi_0$, becomes meaningful in the limit $\rho \to \infty, \alpha \to 0$, with $\rho\alpha$ kept finite. However, as soon as ρ is finite, and α non vanishing, corrections due to disorder should exhibit non trivial magnetic impurity signatures, like broadening of Landau levels and localization. In a first approach to this problem, a Brownian motion analysis partially relying on lattice numerical simulations, will exhibit a global shift $\frac{|e<B>|}{2m} =< \omega_c >$ in the Landau spectrum of the average magnetic field. In the case $\alpha = \pm 1/2$, on the other hand, where the disorder is clearly non perturbative, a depletion of states at the bottom of the spectrum will manifest

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itself by a Lifschitz tail in the average density of states. Secondly, a quantum mechanical formulation will be used, where short range singular A-B interactions are properly taken into account by a wave function redefinition, allowing for an analytical averaging on the disorder. If one assumes that the total average magnetic field $B+ < B >$ is strong enough so that one can neglect the coupling between the Lowest Landau Level (LLL) and the excited Landau levels by the random component of the vortex distribution, the system will be best described when projected in the LLL. Since one has in view a sufficiently dilute gas of electrons compared to the available quantum states in the LLL — the fractional Quantum Hall regime —, such a restriction is licit. It will be explicitly mapped on an equivalent problem of random δ impurities, from which additional information on Landau levels broadening will be extracted.

2. Brownian Motion

One starts from a square lattice of $\mathcal{N}$ squares of size a^2, in which point magnetic impurities are randomly dropped. Let N_i be the number of vortices dropped on square i. A random configuration $\{N_i\}$ will be realized with the probability

$$P(\{N_i\}) = \frac{N!}{\mathcal{N}^N \prod_{i=1}^{\mathcal{N}} N_i!} \xrightarrow[\mathcal{N}\to\infty]{} \prod_i^{\mathcal{N}} \frac{(\rho a^2)^{N_i} \mathrm{e}^{-\rho a^2}}{N_i!} \tag{1}$$

with $N/\mathcal{N} = \sum_{i=1}^{\mathcal{N}} N_i/\mathcal{N} = \rho a^2$. In order to compute the average level density $< \rho(E) >$, one focuses, in the thermodynamic limit $\mathcal{N} \to \infty$, on the one-electron partition function

$$Z = Z_0 < \mathrm{e}^{i \sum_{i=1}^{\mathcal{N}} 2\pi n_i N_i \alpha} >_{\{C, N_i\}} \tag{2}$$

where $\{C\}$ is the set of L steps closed random walks, and n_i is the number of times the square i has been wound around by a given random walk in $\{C\}$, i.e. its winding number. $Z_0 = \frac{1}{2\pi t}$ is the free partition function, with t the length of the curve ($2t = La^2, e = m = 1$). (2) is invariant when α is shifted by an integer, so one can always restrict to $|\alpha| < 1$. To get a continuous path integral formulation for Brownian paths in the plane, one simply considers the limit $a \to 0, L \to \infty$, with t finite. Averaging Z with (1) one gets

$$Z = Z_0 < \mathrm{e}^{\rho \sum_n S_n (\mathrm{e}^{i 2\pi\alpha n} - 1)} >_{\{C\}} \tag{3}$$

where S_n stands for the arithmetic area of the n-winding sector of a given Brownian path in $\{C\}$. Consider the limit of no disorder, $\alpha \to 0, \rho \to \infty$, and $< B >$ finite. One expects that $Z \to Z_0 < \mathrm{e}^{i<B>\sum_n n S_n} >_{\{C\}}$, the partition function of one electron in an uniform magnetic field $< B >$. However, possible corrections coming from the exponent $\exp(i2\pi n\alpha) - 1$ might alter this result. Due to the non-differentiability of Brownian paths, $\sum_n n^2 S_n$ is not defined for a typical Brownian curve where $< S_n >= \frac{t}{2\pi n^2}$ [2]. It has been shown [3] that S_n is strongly peaked when $n \to \infty$, namely the probability distribution of the variable $x = n^2 S_n$ tends to $\delta(x - t/2\pi)$, with a variance smaller than a constant times $n^{-\eta}, \eta = 18$. Also, S_n and $S_{n'}$ are very weakly correlated when n and n' become large. Since one wants to compare the partition function Z of the electron to its partition function in the average magnetic field $< B >$, one can as well consider the partition function Z_B of the electron not only subject to the random vortices, but also to an uniform magnetic field B,

$$Z_B = Z_0 < \mathrm{e}^{-2<B>S} \cos(< B > \mathcal{A}) >_{\{C\}} \tag{4}$$

where the variables $\mathcal{S}$ and $\mathcal{A}$ are defined as

$$\mathcal{S} = \frac{1}{2\pi\alpha}\sum_n S_n \sin^2(\pi\alpha n); \quad <\mathcal{S}> = \frac{\alpha}{|\alpha|}\frac{t}{4}(1-|\alpha|) \tag{5}$$

$$\mathcal{A} = \frac{1}{2\pi\alpha}\sum_n S_n(\sin(2\pi\alpha n) + 2\pi\alpha n\frac{B}{<B>}); \quad <\mathcal{A}> = 0 \tag{6}$$

and, in the case $B = -<B>$, compare $Z_{-<B>}$ to Z_0. For averaging (4) in $\{C\}$, the probability distributions of $\mathcal{S}$ and $\mathcal{A}$ are needed. If one splits $\sum_n = \sum_{|n|\le n'} + \sum_{|n|>n'}$, with n' sufficiently large so that the x probability distribution can be used when $|n| > n'$, one gets

$$P(\mathcal{S}) \xrightarrow[\alpha\to 0]{} \delta(\mathcal{S} - \frac{\alpha}{|\alpha|}\frac{t}{4}); \quad P(\mathcal{A}) \xrightarrow[\alpha\to 0]{} \delta(\mathcal{A}) \tag{7}$$

One deduces that, in the limit $\alpha \to 0$, $Z_{-<B>} \xrightarrow[\alpha\to 0]{} Z_0 \mathrm{e}^{-t|<B>|/2}$, implying that the system of random vortices is equivalent to an uniform magnetic field $<B>$, but with an additional positive shift $|<B>|/2 = <\omega_c>$ in the Landau spectrum. The origin of this shift can be traced back to the S_n Brownian law: when one counts probability of winding around fixed points *via* A-B path integral technics [2], the quantum A-B particle is forbidden to coincide with the vortex location, *via* contact repulsive interactions. This procedure [4] has deep implications, both in the A-B and in the anyon contexts (many-body A-B problem), in particular for Bose-based perturbative computations. One will come back to this point in the sequel.

So far, one has considered the average magnetic field limit $\alpha \to 0$. Clearly, the analysis for intermediate values of α becomes quite involved. In the $\alpha = \pm 1/2$ case, one can explicitly test the effect of the random distribution of vortices. (3) now reads $Z = Z_0 < \mathrm{e}^{-2<B>\mathcal{S}} >_{\{C\}}$ where $\mathcal{S} = \sum_{n \text{ odd}} S_n/\pi$ is a well-behaved random variable which, because of the probability distribution law for the S_n's, scales like t. Thus the $y = \pi\mathcal{S}/t$ probability distribution $P(y = \pi\mathcal{S}/t)$ can be obtained by simulations on a lattice, where a number of steps ranging from 2000 to 32000 has been used. The average level density, obtained by inverse Laplace transform of Z, is found to be a function of E/ρ only,

$$<\rho(E)> = \rho_0(E)\int_0^{\frac{E}{2\rho}} P(y = \pi\mathcal{S}/t))\mathrm{d}y \tag{8}$$

More generally, it is easy to convince oneself that the inverse Laplace transform of the partition function Z_B given in (4) is necessarily a function of $E/\rho, B/\rho$ and α. In Figure 1, $<\rho(E)>$ displays a Lifschitz tail at the bottom of the spectrum, around $E \simeq 2\rho <y> = \pi\rho/4$, where a behavior $<\rho(E)> \simeq \exp(-\rho/E)$ is expected. The energy level depletion at the bottom of the spectrum is coherent with the positive shift in the Landau spectrum of the average magnetic field. It is also reminiscent of the singular A-B density of states depletion $\rho(E) - \rho_0(E) = \frac{\alpha(\alpha-1)}{2}\delta(E)$ [5] ($0 < \alpha < 1$ is understood). Since, the average density of state happens to depend only on the scaling variable E/ρ, for a given impurity density, a transition from a Lifschitz tail density of states to a Landau density of states is expected when α becomes sufficiently small ($\alpha_c \simeq 0.3 - 0.4$) [6]. Finally, (3) generalizes to the case of m species of vortex α_i with density ρ_i, $Z = Z_0 < \prod_{i=1}^m \mathrm{e}^{\rho_i \sum_n S_n(\mathrm{e}^{i2\pi\alpha_i n}-1)} >_{\{C\}}$. In the limit $\alpha_i \to 0$, with $\rho_i\alpha_i$ finite, when $\sum_{i=1}^m \rho_i\alpha_i = 0$, no average magnetic field materializes, and only disorder effects are expected. Similarly, a mixture of $\pm\alpha$ vortices with same density exhibits only a Lifschitz tail density of states.

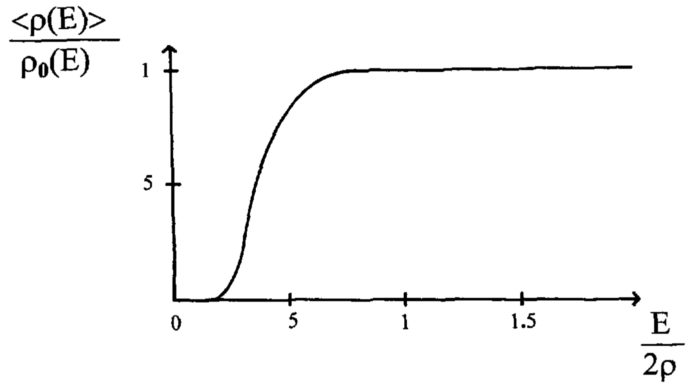

Fig. 1. — The average level density of states $< \rho(E) >$ as a function of the variable $E/(2\rho)$ ($\alpha = \pm 1/2$, no external field) exhibits a Lifshitz tail at the bottom of the spectrum.

3. Operator Formalism

The Hamiltonian of an electron in a constant magnetic field $\mathbf{B} = B\mathbf{k}$ ($\mathbf{k}$ is the unit vector perpendicular to the plane, $|\alpha| < 1$, $eB > 0$ has been assumed without loss of generality), superposed onto N vortices reads

$$H = \frac{1}{2m}\left(\mathbf{p} - \sum_{i=1}^{N} \alpha \frac{\mathbf{k} \times (\mathbf{r} - \mathbf{r}_i)}{(\mathbf{r} - \mathbf{r}_i)^2} - \frac{eB}{2}\mathbf{k} \times \mathbf{r}\right)^2 + \sum_i \frac{\pi|\alpha|}{m}\delta^2(\mathbf{r} - \mathbf{r}_i) \tag{9}$$

In the presence of B, the sign of α becomes a physical observable, i.e. the interpolations $\alpha : -1/2 \to 0$ and $\alpha : 0 \to 1/2$ are not symmetric with respect to $\alpha = 0$, or, in other terms, the limits $\alpha \to 0^+$ and $\alpha \to 0^-$ differ. In order to illustrate this statement, consider the degenerate ground state (energy $\omega_c = eB/2m$) [7]

$$\psi_0(\mathbf{r}) = \prod_i |z - z_i|^{m_i - \alpha} e^{i m_i \arg(z - z_i)} e^{-\frac{1}{2} m \omega_c z \bar{z}} \qquad m_i \geq \alpha \tag{10}$$

The eigenstates (10) vanish when the electron coincide with a magnetic impurity. This restriction means unpenetrable vortices, a hard-core boundary prescription that has already been related, in the Brownian motion approach, to the positive shift in the Landau spectrum. In the Hamiltonian formulation (9), it is given a precise meaning by adding to the pure Aharonov-Bohm Hamiltonian the contact repulsive interactions. Of course, once the Hilbert space (10) has been defined, the δ functions trivially vanish on this space. They amount to couple the spin 1/2 degree of freedom [8] of the test particle, endowed with a magnetic moment $\mu = -\frac{e}{2m}\alpha/|\alpha|$ (thus an electron with gyromagnetic factor $g = 2$), to the infinite magnetic field inside the flux-tubes. An important consequence is that the ground state basis (10) is complete when $\alpha < 0$, since one obtains the complete LLL basis of the B field in the limit $\alpha \to 0^-$ ($m_i \geq 0$). On the contrary, when $\alpha \to 0^+$ ($m_i > 0$), N excited states, which are not analytically known, merge in the ground state to yield a complete LLL basis [7]. Clearly, when $\alpha \to 1$, these excited states have to become usual excited Landau levels, simply because the system is periodic. This pattern for the excited states is crucial for the analysis that follows.

Note that in the 1-vortex case, which is entirely solvable, the spectrum is composed of an infinitely degenerate Landau spectrum $(2n+1)\omega_c$ and a $n+1$ degenerate spectrum $(2n+1+2\alpha)\omega_c$, which interpolates between the Landau levels. This structure will be shown to remain valid in the low density impurity system, at least at the bottom of the spectrum.

To proceed further, one takes into account the behavior of the ground state near the magnetic impurities, quite analogously to what is done in the A-B and anyon perturbative analysis [4]. In the present situation, however, one is rather interested by averaging over the disorder, and by testing the average magnetic field contribution to the Landau spectrum of the electron. One performs the nonunitary transformation

$$\psi(\mathbf{r}) = \mathrm{e}^{-\frac{1}{2}m<\omega_c>r^2}\prod_i |\mathbf{r}-\mathbf{r}_i|^{|\alpha|}\tilde{\psi}(\mathbf{r}) \tag{11}$$

to obtain an Hamiltonian $\tilde{H}$ acting on $\tilde{\psi}(\mathbf{r})$ where the impurity potential reads

$$V(\alpha<0) = \sum_{i=1}^{N}\frac{2\alpha}{m}\frac{\partial_{\bar z}}{z-z_i} + \sum_{i=1}^{N}(\omega_c + <\omega_c>)\alpha\frac{z}{z-z_i} \tag{12}$$

$$V(\alpha>0) = -\sum_{i=1}^{N}\frac{2\alpha}{m}\frac{\partial_{z}}{\bar z-\bar z_i} + \sum_{i=1}^{N}(\omega_c + <\omega_c>)\alpha\frac{\bar z}{\bar z-\bar z_i} \tag{13}$$

The average magnetic field pre-exponential factor in (11) has to be understood in the infinite density limit as compensating for $\prod_i|\mathbf{r}-\mathbf{r}_i|^{|\alpha|} = \mathrm{e}^{\sum_{i=1}^{N}|\alpha|\ln|\mathbf{r}-\mathbf{r}_i|} \to \mathrm{e}^{\rho|\alpha|\int \mathrm{d}^2\mathbf{r}'\ln|\mathbf{r}-\mathbf{r}'|} = \mathrm{e}^{\frac{1}{2}m<\omega_c>r^2}$ Short range singular interactions $\alpha^2/|z-z_i|^2$ present in H, as well as the δ interactions, have been traded off for the regular interactions $|\alpha|\partial_{\bar z}/(z-z_i)$ in $\tilde{H}$. Moreover, interactions involving two magnetic impurities have disappeared from $\tilde{H}$, which will greatly simplify the average on the disorder. The Hamiltonians H' and $\tilde{H}$ are equivalent, and can be indifferently used for computing the partition function or the density of states. More rigorously, consider instead of (11)

$$\psi(\mathbf{r}) = \prod_i\frac{|\mathbf{r}-\mathbf{r}_i|^{|\alpha|}}{<|\mathbf{r}-\mathbf{r}_i|^{|\alpha|}>}\tilde{\psi}(\mathbf{r}), \tag{14}$$

where the average $<>$ is done in a finite volume $V=N/\rho$. If the redefinition (14) affects the short distance behavior of the Hilbert space, it does not modify its long distance behavior, and thus the normalisation of the wave functions. It leads to a redefined Hamiltonian which is appropriate for estimating $<\rho(E)>$ in a functional approach [9].

If one now extracts the mean-value of $V(\alpha)$, one obtains

$$\tilde{H} = <\omega_c> + H_{B+<B>} + V(\alpha) - <V(\alpha)> \tag{15}$$

where $H_{B+<B>} = -\frac{2}{m}\partial_z\partial_{\bar z} + \frac{m}{2}{\omega_t}^2 z\bar z - \frac{e(B+<B>)}{2m}(z\partial_z - \bar z\partial_{\bar z})$ is the Landau Hamiltonian for the $B+<B>$ field. One has added for convenience to $\tilde{H}$ a long distance harmonic regulator $m\omega^2r^2/2$, which partially lifts the degeneracy of the spectrum -one has $\omega_t^2 = (\frac{e(B+<B>)}{2m})^2 + \omega^2$, the thermodynamic limit is obtained by letting $\omega \to 0$.

The global shift $<\omega_c>$ appears explicitly in (15). It is the only remnant effect of disorder in the average field limit $\alpha\to 0, \rho\to\infty$, where $V(\alpha) - <V(\alpha)> \to 0$. If $\alpha<0$, $\tilde{H}$ should be trivially diagonal when restricted to the LLL of the $B+<B>$ field, since the redefined ground state basis $\tilde{\psi}_o$ derived from (11) is identical (the excited states play no role at all) to the LLL basis $< u|z >= u(z)\exp(-m\omega_t z\bar{z}/2)$, with $u(z)$ holomorphic ([1]). One respectively finds, when $\alpha < 0$ and $\alpha > 0$

$$< v|V(\alpha)- < V(\alpha) > |u >=< v|(\omega_c - \omega_t)\alpha z(\sum_{i=1}^{N} \frac{1}{z - z_i} - \pi\rho\bar{z})|u > \tag{16}$$

$$< v|V(\alpha)- < V(\alpha) > |u >=< v|(\omega_c - \omega_t)\alpha\bar{z}(\sum_{i=1}^{N} \frac{1}{\bar{z} - \bar{z}_i} - \pi\rho z) + \frac{2\pi\alpha}{m}(\sum_{i=1}^{N} \delta(z - z_i) - \rho)|u > \tag{17}$$

In the thermodynamic limit $\omega \to 0$, (16) vanishes, so that $\tilde{H} = \omega_c$ is indeed diagonal on the LLL basis. More interesting is the case $\alpha > 0$, since then

$$\tilde{H} =< \omega_c > + H_{B+<B>} + \frac{2\pi\alpha}{m}(\sum_{i=1}^{N} \delta(z - z_i) - \rho) \tag{18}$$

Thus, the magnetic impurity problem, when projected on the LLL of $B+ < B >$, is mapped on a δ impurity problem ([2]), which encodes the effect of the N excited states that leave the ground state when $\alpha \to 0^+$. The contact interactions pattern happens to be valid for vortex systems in general, as soon as the excited states which join the Landau ground state are properly taken into account. It holds in particular for the N-anyon Hamiltonian $\tilde{H}_N$ which, when projected on the LLL of an external B field, becomes $\tilde{H}_N = \sum_{i=1}^{N} H_B(\mathbf{r}_i) + \frac{4\pi\alpha}{m}\sum_{i<j}\delta(\mathbf{r}_i - \mathbf{r}_j)$.

Projecting on the LLL of a Landau basis is meaningful if one has a magnetic field at disposal, which builds a well separated Landau spectrum for the test particle. When the external magnetic field is strong compared to the average magnetic field, $\rho_L >> \rho|\alpha|$, the $V(\alpha)- < V(\alpha) >$ contribution is small compared to the cyclotron gap. On the other hand, in the regime $\rho_L \simeq \rho|\alpha|$, a physical average magnetic field is needed, namely $\rho \to \infty, \alpha \to 0$, with $\rho\alpha$ finite. In a semi-classical point of view, the number $f = \rho/(\rho_L + \rho\alpha)$ of magnetic impurities enclosed by the classical electronic orbit in the magnetic field $B+ < B >$ has to be big ($\rho_L = eB/2\pi$ is the Landau degeneracy). If this is the case, $V(\alpha)- < V(\alpha) >$ does not couple different Landau levels of $B+ < B >$. What is the effect of disorder? One is interested by the average density of states, a local quantity, which can be directly computed in the thermodynamic limit $\omega = 0$. If $\alpha < 0$, the impurity potential vanishes, and disorder has no effect. On the other hand, when $\alpha > 0$, one has to average on δ contact interactions, a problem already studied by Brezin *et al.* [9], motivated by an original study of Wegner [10]. They considered a Poisson distribution (it is nothing but (1) in the thermodynamic limit) of uniformly distributed δ impurities in the LLL of a $\mathcal{B}$ field, with Hamiltonian $H = H_{\mathcal{B}} + \lambda\sum_{i=1}^{N}\delta(\mathbf{r} - \mathbf{r}_i)$. If one now sets $\lambda = 2\pi\alpha/m$, $\mathcal{B} = B+ < B >$, $\nu = \frac{f}{\lambda\rho}(E - \omega_c)$, one finds [9] for the average density of states of (18)

$$< \rho(E) >= \frac{1}{\pi\lambda} Im \quad \partial_\nu \ln \int_o^\infty \mathrm{d}t \exp(i\nu t - \epsilon t - f\int_o^t \frac{\mathrm{d}t'}{t'}(1 - \mathrm{e}^{-it'})) \tag{19}$$

where $\epsilon \to 0$ is understood. $< \rho(E) >$ is clearly a function of α, B/ρ and E/ρ.

([1]) One assumes ω_c larger than $< \omega_c >$. If not, an antiholomorphic basis has to be used. However, one can as well consider the opposite situation with $e < B >$ positive, and eB negative, i.e. $< B > +B > 0$, which is in fact described by (17).

([2]) The δ interactions in (18) act non trivially on the Hilbert space of the LLL of $B+ < B >$. They should clearly not be confused with the hard-core boundary δ functions introduced in (9), which do vanish on the groundstate Hilbert space (10).

4. Discussion and Conclusion

If $\alpha < 0$, one trivially gets that for $< \rho(E) >$ a δ peak centered at $E = \omega_c$, with degeneracy $\rho_L + \rho\alpha$. A direct computation, based on (16), of the partition function of the test particle in the presence of a harmonic regulator, leads to the same result, once the thermodynamic limit is taken. For $\alpha > 0$, interesting effects due to disorder are expected. In the regime where the external magnetic field is dominant, where typically $f < 1$, the results read off [9] give a δ peak also centered at $E = \omega_c$, with the same (by periodicity) degeneracy $(\rho_L + \rho\alpha)(1-f) = \rho_L + \rho(\alpha - 1)$ and ρ additional states (per unit volume) broadening the LLL as $(E-\omega_c)^{-f}$ These states exactly correspond in the thermodynamic limit to the $N = \rho V$ excited states leaving the groundstate. $f = 1$ is critical because the LLL is then entirely depleted by the excited states (the degeneracy of the LLL grows as $\rho\alpha$, but the depletion grows as ρ). One understands the bump in the density of states observed at $\nu = 1$, i.e. at energy $E = \omega_c + \frac{2\pi\alpha}{m}(\rho_L + \rho\alpha) = \omega_c(1+2\alpha+2\alpha^2\rho/\rho_L) \simeq \omega_c(1+2\alpha)$ at low impurity density (see the 1-vortex case described above), as the excited states contribution to the density of states. When $\alpha \to 1$, where a usual Landau spectrum has to be recovered, one indeed finds $E = 3\omega_c$, to a correction of order ρ/ρ_L, a manifestation of the approximation made when projecting on the LLL (note that one necessary has $\rho_L >> \rho\alpha \simeq \rho$, thus $\rho/\rho_L \simeq 0$). So, most of the ρ excited states join the first excited Landau level of the B field when $\alpha = 1$ (the others excited states join the Landau spectrum at higher levels).

In the average magnetic field regime, where f has to be big, the shift in the spectrum can be directly understood from (19). One has $\nu \to \infty$, therefore only small values of t contribute to the oscillating term, implying that the integral $\int \mathrm{d}t'(1-\mathrm{e}^{-it'})/t' \simeq it$. One gets a Landau density of states for the average magnetic field with the shift $\nu \to \nu - f$, that is to say the desired $E \to E - 2 \times \frac{e<B>}{2m}$ In the particular case $B = 0$, the magnetic field is entirely due to the magnetic impurities. Then, the parameter $f = 1/\alpha$ depends only on the A-B coupling constant.

In conclusion, an open question concerns the conductivity [11] properties of an electron in the presence of magnetic impurities. In the Quantum Hall regime, information about localisation properties of the eigenstates would be of great interest. Moreover, the test particle has been shown to satisfy a kind of exclusion principle with the magnetic impurities, *via* contact repulsive interactions. It would be more satisfactory if statistical effects could be also encoded in the distribution of the impurities themselves. Concerning the Brownian motion approach, the random magnetic field problem (windings) and the δ impurity problem (excursions) are, *a priori*, very different. The fact that they coincide when one considers their projection on a LLL certainly deserves further explanations.

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